Our students’ relationships with mathematics—and our own relationships, for that matter—are far more complicated than whether we like the subject or are good at it. For example, you may have known students who were good at math and quite confident in their mathematical abilities who wanted nothing more than to...
take their final math class and be done with it. Or consider the student who struggles in class, takes longer than most to understand the concepts, but who perseveres and eventually goes into a math-related field. Why does one student think it is worth the effort to continue in math when the other does not?

We discuss some of the dispositions that comprise students’ (and teachers’) relationships with mathematics—what we call a mathematical identity. We present tasks that can be used with math students at all levels to help them and us become more aware of our mathematical identities. Our premise is that although many similarities are likely among our students’ mathematical identities, there are also many differences. These differences may not be apparent in the classroom or even in individual conferences with students. In fact, most of us—students and teachers alike—have never tried to articulate these beliefs.
Reflect and Discuss

Reflective teaching is a process of self-observation and self-evaluation. It means looking at classroom practice, thinking about what is done and why, and then evaluating whether it works. Collecting information about what goes on in the classroom and then analyzing and evaluating this information will allow teachers to identify and explore their practices and underlying beliefs.

The following questions are suggested prompts to help you, the reader, reflect on the article and on how the authors’ ideas might benefit your classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

- What can placement of words in the descriptive words task (see fig. 1) reveal about students’ beliefs concerning what it means to be good at mathematics?
- Take a moment and consider where you would place yourself on the continua in figure 3. Where would you place your fellow teachers?
- Where would you place your students? Your best students? Where do you think they would place themselves?
- What might particular placements on this continuum say about an individual’s mathematical identity?

You are invited to tell us how you used “Reflect and Discuss” as part of your professional development. The Editorial Panel of Mathematics Teaching in the Middle School appreciates the interest and values the views of those who take the time to send us their comments. Send letters to mtms@nctm.org, and include “Readers Write” in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit may be subject to abridgment. Letters are also edited for style and content.

We encourage readers to think about their own beliefs as they read this article. A greater awareness of our own beliefs about math, and of the range of different beliefs our students may have, can significantly influence classroom interactions and help us motivate students.

MATHEMATICAL IDENTITY

We define mathematical identity as an individual’s relationship with mathematics. That is, the ways a person learns, does, thinks about, retains, or chooses to associate with the subject. Many aspects of a person’s mathematical identity may be observable; we can see that someone chooses to study (or not to study) mathematics, and we can see how he or she performs on a test or homework. However, underlying our relationship with mathematics is a system of dispositions about mathematics that are not so easily observed. We consider the relationship that one displays with math as a manifestation of the dispositions one holds about math—dispositions about the subject itself and one’s ability to do it.

Three dimensions can help define our relationships with math. For each dimension, we discuss a task and pose questions that can be used to become more aware of students’ mathematical identities. The task can help you, the reader, think about your own beliefs. Typical responses that students have given to these questions demonstrate the range of beliefs that students hold. We conclude by discussing how this knowledge is beneficial to teachers.

WHAT IT MEANS TO BE GOOD AT MATH

How does an individual know if he or she is good at math? What kinds of people are good at math? Although you may have never explicitly answered these questions, you most likely have beliefs about what it means to be good at mathematics. In our discussions with
prealgebra, beginning algebra, and even college calculus students, we have used a descriptive words task (see **fig. 1**) to elicit students’ beliefs about what it means to be good at math.

Many students think about mathematics very narrowly (at least explicitly) and do not associate words such as *creative* or *obedient* with this topic. We decided to force the issue to see just what associations students made with words not typically used to describe math. We tried to choose words that would elicit a wide variety of responses and thus reveal differences both stark and subtle in students’ mathematical identities. We often use this task in a whole-class setting by displaying the number line, choosing one word to focus on, and then asking students to raise zero to five fingers according to where they would place that word on the number line. We then invite students to share why they ascribed various positions to the number.

To see how such conversations play out, we discuss three words from the list: creative, obedient, and social. We begin by asking some follow-up questions that we have found useful in our own class discussions. As you read through these questions and some of the typical responses we have received, consider your own beliefs about math as well as the various beliefs you have observed with your own students.

**Creative**
- Is it important for a person to be creative to do well at math? How important?
- Is it necessary to be creative or merely helpful?
- Could a person be even better at math if he or she were creative?
- What does it mean to be creative?
- Are you creative?
- Would the same kind of creativity be found in the arts as compared with math?
- What is different?

A wide variety of responses emerge in classroom discussions about the relationship between math and creativity. Invariably some claim that you do not need to be creative to do mathematics; instead, you need to know how to follow instructions. By contrast, one prealgebra student volunteered that when she gets help with story problems, it always seems that she needed to think about the question differently. She concluded that this different way of thinking could be termed *creativity*, and that her lack of it was the reason that she struggled. In our experience, many students have never considered the relationship between math and creativity.

**Obedient**
- Would someone who is good at mathematics be obedient?
- Is it desirable to be obedient?
- Should someone be obedient all the time?
- Are there times when obedience would hinder your success with math?
- Does obeying the rules of math make you good at it?

For many people, obedience within a discipline has negative connotations. Boaler and Greeno (2000) found that students who believed that math merely concerned following the rules were less likely to want to continue in math, even though they had been successful in their classes. In discussions with prealgebra students about obedience in the subject, some cited the following as an example of the rules in math: The rule $2 + 2$ always had to equal 4. Looking at it that way, there are a lot of rules. No wonder students feel overwhelmed by all the rules they have to learn and obey.

**Social**
- Is math a topic best learned in a group or individually?
- Is it better to listen to the teacher individually, or is there a benefit to discussing mathematics?
- Is it ever helpful to listen to other people’s ideas or verbalize your own ideas about mathematics?
- Are study groups helpful?
- What should be your role in a study group?
- If you already know how to do a problem, do you receive any benefit by explaining it to others?
- If someone is good at math as an individual, would participating in a group discussion make him or her better?

Teachers know from experience that when you teach a concept, you come to understand it better; many students, however, have yet to learn this important principle. They often suppose it is only those who really understand math who can explain it. They have not considered that
providing explanations could be part of the learning process. Once students are in a position to talk about and explain certain concepts, they begin to see the benefits of such interactions.

One student from our research, who had recently experienced a discussion-oriented math classroom for the first time, said he needed someone to talk to and bounce ideas off. He gave the example of preparing for a test and getting stuck on a concept. He called one of his classmates and went to her house. Even though she did not explain any concepts to him, he said, “As I started talking to her, I could visualize what my problem was and how then to do the problems.”

Discussions about the degree to which terms such as creative, obedient, and social describe people who are good at math begin to reveal important aspects of individuals’ mathematical identity. Several words included in this task are meant to stimulate discussion about views of math that can limit learning it. For example, the word obedient elicits conversations about memorization and following rules. The word brilliant describes the degree to which all students can be good at math. One student in our research felt she was pretty good at the subject, was in an honors calculus class, and yet rated brilliant as a 5 in this task. She said a person would have to be brilliant to really be good at math. She felt she could work really hard and do okay but she would never be really good at it because she was not brilliant.

We have found that students who participate in such discussions generally see at least a few of these attributes in themselves. In fact, research has shown that people have very different ideas about mathematics (Boaler and Greeno 2000; Kloosterman, Raymond, and Emenaker 1996).

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Fig. 2 The mathematical categories task can be a valuable tool to help students consider how they themselves define mathematics.

| Arranging trophies from tallest to shortest | Sorting shapes | Cooking with a recipe | Memorizing times tables |
| Playing Sudoku | Arguing a case before a jury | Quilting | Playing a video game |
| Playing the drums | Driving a car | Drawing a map | Playing the piano |
| Reading a book | Reading a map | Keeping a planner | Writing an essay |
| Solving a scheduling problem | Playing solitaire | Doubling a recipe | Balancing your checkbook |
| Working on an electrical circuit | Hanging a picture on the wall | Playing a video game | Riding a skateboard |

WHAT IS MATHEMATICS? HOW USEFUL IS IT?
What exactly is mathematics? Is it only found in the classroom? Who uses it? How often? Most of us do not go around defining mathematics. Pause for a moment, and try to define it. It is not as easy as it sounds to come up with a complete definition. In fact, research has shown that people have very different ideas about mathematics (Boaler and Greeno 2000; Kloosterman, Raymond, and Emenaker 1996).

The mathematical categories task (see fig. 2) is a valuable tool that can help students consider how they themselves define mathematics. If asked to name activities that use mathematics, students would probably list the obvious choices such as...
engineering or science. For those who are not in those fields, math may seem somewhat useless. In this task, we ask students to consider what if any math is used in many different activities, some of which they likely have never associated with it. We tried to include many obvious and not-so-obvious activities. We also listed examples designed to bring out different aspects of math like logic (arguing a case), geometry (quilting, sorting shapes), or patterns (arranging trophies, tiling the bathroom).

We often print out these twenty-eight activities on cards and ask students to sort the cards according to their own categories. How might you arrange these activities in mathematical categories? In determining your categories, consider what distinguishes one category from another. These questions can be considered when discussing some of the twenty-eight activities:

- Is comparing the sizes of trophies a mathematical activity if you do not use numbers?
- Would Sudoku be mathematical if we used letters or symbols instead of numbers?
- Is finding or using patterns part of mathematics?
- Is a lawyer who is proving a case to a jury using the same skills as those found in geometry?

Students define what is mathematical in a variety of ways. Some believe numbers are essential. One student said arranging trophies by size was purely aesthetic. Another mentioned that a lawyer uses mathematics only when a particular case uses numbers, such as the speed of a car or the amount of money to be awarded.

However, some students have much broader definitions of mathematics. One student categorized mathematics as either recognizing patterns or manipulating patterns. “Reading a book” was listed under recognizing a pattern, and “writing an essay” was part of manipulating the pattern.

Beliefs about the nature of math affect students’ attitudes toward it. Students often think of mathematics as “that stuff you do during your math class.” When they have a negative experience in a class or have trouble understanding a concept, they believe it is “math” they hate. They do not see the mathematics in other aspects of their lives. After having a discussion such as you might have with this task, we as teachers can challenge students to be more specific when they express a like or dislike for the subject. For example, during a discussion about this task, one student admitted that she dislikes mathematics only when she does not understand a concept. Although no one likes to be in that
position, students often want to discard the entire topic because of one bad experience.

As students broaden their views about the nature of math, they begin to see its value in many different activities and professions. Students are often surprised to learn, for example, that a bachelor’s degree in mathematics is considered to be excellent preparation for law school.

BELIEFS ABOUT ABILITY AND DESIRE
The importance of one’s confidence and self-efficacy with regard to mathematics has been discussed at length in the literature (i.e., Bandura 1993; Bandura, Barbaranelli, and Caprara 1996; Forster 2000; Pajares and Miller 1994; Schunk and Pajares 2002). The negative affects of anxiety toward mathematics (i.e., Malinsky et al. 2006; Perry 2004; Tobias 1993) have also been topics of research. However, determining how we feel about our ability to do math is not necessarily straightforward. The words we use to describe our feelings are relative terms. Everyone may feel some anxiety, but is it more than someone else or less than it used to be? We felt we needed some way to help students describe their relationship with this subject.

We designed the continuum task (see fig. 3) as one way to help students become more aware of their feelings. We asked a group of five students who had been working together in a class to fill out the continua on one another. This exercise revealed that some students’ beliefs were not apparent to their peers. For example, everyone in the group placed Caleb at the “peace” end of the anxiety continuum, except for Caleb himself (see fig. 4a). Caleb placed himself in the middle and said that some situations caused him quite a bit of anxiety. In class, he hid his anxiety, and no one—neither students nor teachers—were aware of it.

Also, notice where Hannah was placed by Caleb (see fig. 4b) and by Jack (see fig. 4c). Caleb placed Hannah as the most anxious, while Jack placed her as the least anxious. Such extreme differences in perceptions highlight the complexities of our mathematical identities and how difficult it is for others to perceive them. Discussions about these complexities can help both students and teachers better understand one another and thus better support one another as we seek to develop healthy relationships to mathematics.

As another example, Bryce often showed a lot of confidence in class. He was known to have been so confident that he would convince others to do a problem a certain way, even when it was wrong. Most students placed him pretty high on the confidence continuum; however, he himself did not feel confident. Bryce’s overall self-confidence masked his lack of confidence regarding mathematics.

Participating in this continuum task helped to reveal this hidden aspect of this student’s mathematical identity to the researchers, to the teachers, and to Bryce. Again, we emphasize that the value in carrying out this continuum activity comes not from documenting our positions on the continua but from trying to articulate our reasons for choosing

![Fig. 4 Students often placed themselves at different points along the continuum as compared with how their classmates viewed and placed them.](image-url)
these positions. We have found that students often place themselves in quite different places than we would place them or in similar places but for reasons that differ greatly from our suppositions.

CONCLUSION
As teachers, we must keep in mind that the mathematical identity one sees with a cursory glance may be misleading. Our mathematical identities are both complex and diverse. Some beliefs are not readily apparent, and a student’s behavior can actually serve to disguise rather than reveal a person’s mathematical identity.

Tasks such as those described here can help teachers as well as students become more aware of their relationships with math. (Other related and worthwhile tasks can be found in work that focuses on students’ beliefs [e.g., Spangler 1992] and on multiple intelligences [e.g., Teele 1997].) Simply participating in these tasks has several immediate benefits:

• When students hear other students express beliefs about math that differ from theirs, they are more likely to expand their own beliefs.

• These tasks open doors for further discussions. We often find opportunities to talk about different ways that people prefer to do math.

During our classes, we make sure we ask about the different ways that students think about a problem. Such discussions reinforce the idea that there is room for creativity. When students are aware of other ways of thinking, they listen more closely to questions and comments that are made in class.

As we become more aware of our own mathematical identities as teachers, and more aware of the diversity of our students’ identities, we start to see how seemingly innocuous classroom decisions can influence our students’ dispositions toward math. Helping our students become more aware of their mathematical identities can empower them to make that relationship more meaningful, thus motivating them to engage in mathematics in our classrooms and beyond.

REFERENCES


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